

[Last week](#), we discussed a couple of variations of inequality questions with many factors. Let's now look at some more complications that we should know how to handle.

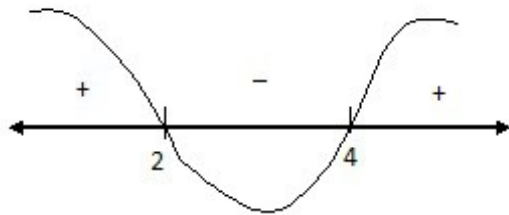
Complication No 3: Even powers

$$(x - a)^2(x - b)(x - c)(x - d) > 0$$

How would you handle even powers of factors? Say, if the question has a factor $(x - a)$ which has an even power, would you still plot 'a' on the number line? No, you will just ignore that factor while making the number line! Why? This is so because this factor will never be negative. It will be either 0 (in case the inequality includes the equality sign e.g. $(x - a)^2(x - b)(x - c)(x - d) \geq 0$) or it will be positive. Therefore, a factor with an even power acts just like a positive constant. Then, does it mean factors with even powers have no role to play at all? No, it doesn't. While writing out the range, they could impact the final answer. We will discuss this in more detail using an example later on.

e.g. $(x - 4)(x - 2)(x + 8)^4 < 0$

You don't need to plot -8 here. Since $(x + 8)^4$ can never be negative, it doesn't change the sign of the expression.



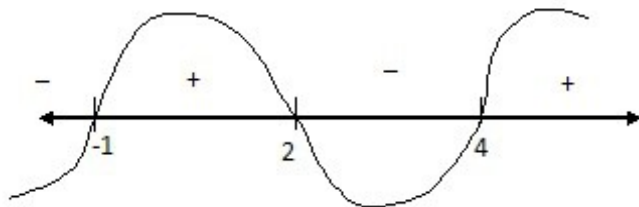
The expression will be negative in the range $2 < x < 4$.

Complication No 4: Odd powers

$$(x - a)^3(x - b)(x - c)(x - d) > 0$$

What will you do in case of odd powers? Notice that in the last two posts, we have handled questions where the power of all the factors is 1. 1 is an odd power. So when you have any other odd power, you will handle it the same way. You can assume that the odd power is equal to 1 and proceed as usual. This is so because the sign of $(x - a)$ will be the same as the sign of $(x - a)^3$.

e.g. $(x - 4)(x - 2)(x + 1)^3 < 0$



The expression will be negative in the range $x < -1$ or $2 < x < 4$.

Now let's look at a question involving both these complications.

Question: Find the range of x for which the given inequality holds.

$$[(2x - 5) * (6 - x)^3] / x^2 \leq 0$$

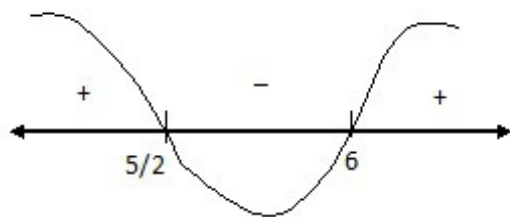
Solution:

I hope you agree that it doesn't matter whether the factors are multiplied or divided. We are only concerned with the sign of the factors.

$$[2(x - 5/2) * (-1)(x - 6)^3] / x^2 \leq 0 \text{ (take 2 common)}$$

$$[2(x - 5/2) * (x - 6)^3] / (x - 0)^2 \geq 0 \text{ (multiply both sides by -1)}$$

Now let's draw the number line. We don't need to plot 0 since $(x - 0)$ has an even power.



We want to find the range where the expression is positive. The required range is $x \leq 5/2$ or $x \geq 6$. But we are missing something here. $x \leq 5/2$ implies that all values less than $5/2$ are acceptable but note that x cannot be 0 since x^2 is in the denominator. Hence the acceptable range is $x \leq 5/2$ or $x \geq 6$ but $x \neq 0$.

When you have the equal to sign, you have to be careful about the way you choose your range.